

Positive Predictive Value based dynamic K -Nearest Neighbor

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Abstract. The K Nearest Neighbors classification method assigns to an unclassified observation the class which obtains the best results after a voting criteria is applied among the observation's K nearest, previously classified points. In a validation process the optimal K is selected for each database and all the cases are classified with this K value. However the optimal K for the database does not have to be the optimal K for all the points. In view of that, we propose a new version where the K value is selected dynamically. The new unclassified case is classified with different K values. And looking for each K how many votes has obtained the winning class, we select the class of the most reliable one. To calculate the reliability, we use the Positive Predictive Value (PPV) that we obtain after a validation process. The new algorithm is tested on several datasets and it is compared with the K -Nearest Neighbor rule.

Keywords: Nearest Neighbor, Supervised Classification, Machine Learning, Non-parametric Pattern Recognition

1 Introduction

In supervised classification problems [1] there are two extremes of knowledge which the modeler may consider. Either (s)he may have complete statistical knowledge of the underlying joint distribution of the observation x and the category θ , or (s)he may have no knowledge of the underlying distribution except that which can be inferred from samples. In the first extreme, a standard Bayes analysis will yield an optimal decision procedure and the corresponding minimum (Bayes) probability of error classification R^* . In the other extreme, a decision to classify x into the category θ is allowed to depend only on a collection of n correct samples $(x_1, \theta_1), (x_2, \theta_2), \dots, (x_n, \theta_n)$, and the decision procedure is by no means clear. This problem is in the domain of supervised classification, and no optimal classification procedure exists with respect to all underlying statistics.

If it is assumed that the classified samples (x_i, θ_i) are independently identically distributed according to the distribution of (x, θ) , certain heuristic arguments may be made about good decision procedures. For example, it is reasonable to assume that observations which are close together (in some appropriate

distance metric) will have almost the same posterior probability distributions on their respective classifications.

Thus to classify the unknown sample x we may wish to weigh the evidence of the nearby x_i 's most heavily. Perhaps the simplest non-parametric decision procedure of this form is the nearest neighbor (NN) classification method, which assigns to x the category of its nearest neighbor.

The first formulation of a rule of the NN type and primary previous contribution to the analysis of its properties is presumed to have been made by Fix and Hodges [2]. They investigated a method that is known as K Nearest Neighbors (K -NN), which assigns to an unclassified point the class most heavily represented among its k nearest neighbors.

In this paper we present a new version of K -NN method which finds the most reliable K for each new case. In a validation process we calculate the success rate when a class is predicted for different K values and for different amounts that belongs to the predicted class between these K Nearest Neighbors. Thus, when a new case has to be classified, we will select the K which get the most reliable result.

This paper is organized as follows. In section 2 we review the K -NN classification method while section 3 is devoted to related work in distance-based classifiers. The new proposed method is introduced in section 5. Section 6 shows the experimental results obtained and in the final section concluding remarks are presented.

2 The K -NN Classification Method

Let $\mathbf{x}_1, \dots, \mathbf{x}_n$ be a correctly classified sample in classes $\theta_1, \dots, \theta_M$, where \mathbf{x}_i takes values in a metric space upon which a distance function d is defined. We will consider the pairs (\mathbf{x}_i, θ^i) where \mathbf{x}_i is the p -variate observation upon the i th individual and θ^i is the class or category which that individual belongs to. We usually say that " \mathbf{x}_i belongs to θ^i " when we mean precisely that the i th individual, upon which measurements \mathbf{x}_i have been observed, belongs to category $\theta^i \in \{\theta_1, \dots, \theta_M\}$.

Consider a new pair (\mathbf{x}, θ) , where only the measurement \mathbf{x} is observable, and where we estimate θ by using the information contained in the set of correctly classified points. We shall call \mathbf{x}' the *nearest neighbor* (NN) of \mathbf{x} if

$$\min_{i=1, \dots, n} d(\mathbf{x}_i, \mathbf{x}) = d(\mathbf{x}', \mathbf{x}). \quad (1)$$

The NN classification decision method gives to \mathbf{x} the category θ^i , precisely the category of its nearest neighbor \mathbf{x}_i . In case of a tie between several neighbors, it has to be broken by modifying the decision rule.

An immediate extension to this decision rule is the so called K -NN approach [3], which assigns the candidate \mathbf{x} the class which is most frequently represented among the k nearest neighbors of \mathbf{x} . In Figure 1, for example, the 3-NN decision rule would decide the class θ_o is active because two of the three nearest neighbors of \mathbf{x} belong to class θ_o .

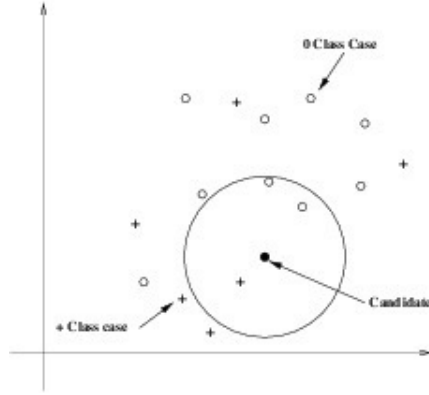


Fig. 1. Third Nearest Neighbor Decision Rule

3 Related work

Much research has been devoted to the K -NN rule [4]. One of the most important results is that K -NN has very good asymptotic performance. Broadly speaking, for a very large design set, the expected probability of incorrect classifications (error) R achievable with K -NN is bounded as follows:

$$R^* < R < 2R^* \quad (2)$$

where R^* is the optimal (minimal) error rate for the underlying distributions. This performance, however, is demonstrated for the training set size tending to infinity, and thus, it is not really applicable to real world problems in which we usually have a training set of hundreds or thousands cases, too few for the number of probability estimations to be performed.

Some distance based approaches, such that of Weinberger et al. [5] try to increase the obtained accuracy in distance based classification by looking for a specific distance, in an automatic way, for each classification problem. The proposed approach could be used to deal with unbalanced or biased databases; a similar idea can be found in other distance based methods [6]. On the other hand, *PEBLIS* instance based inducer (Cost and Salzberg [7]) incorporates MVDM distance metric to deal with symbolic features, a modification of Stanfill and Waltz's VDM metric [8].

Improvement in classification can also be obtained by selecting and/or weighting features (see [9] for an example). Probabilistic voting approaches have also been used ([10], [11]); the main idea here is that each case among the K nearest ones make a weighted vote in favor of the class it belongs to, being the weight the probability each case has to belong to its own class. A very well known approach is the so called Instance Based Learning (IBL), based on the work of Aha [12] and Wettschereck [13]; there are several versions of the algorithm [14].

Another problem that arises with distance-based classifier systems is the management of large databases. Works devoted to data reduction [15] show interesting approaches which could also be used in combination with any other distance based algorithm when the size of the database is significant; other works in this field try to accelerate the execution of the distance based algorithm ([16], [17]) to obtain faster classifications.

Additionally, there are distance based classifiers which aim to deal with the so called multi labeling problem [18], in which, given a new case to be classified, a different number of categories could be given to it. For instance, and taking as example the document categorization area, a newspaper article relating the wedding of some country president would obtain Politics and Society as category labels, being both adequate for the document.

4 PPV K-NN

The standard procedure to use the K -NN algorithm with a new database corresponding to a classification problem, is to first select the appropriate K value for the datafile ($K=5$, for instance) and fix this value to deal with all the cases that have to be classified. The way the most suitable K value is obtained is mainly based on a validation process, although there are other possibilities found in the literature, for example Wang et al. [19] proposed a new method that dynamically adjust the number of nearest neighbors based on the statistical confidence.

As in Wang et al. work, in this paper we propose a new approach to determine the most likely value of K for each of the new cases to be classified. To do that, a validation approach is used as well, not to select a fixed K parameter for all the cases, but to select which K values are best for each of the different categories of the classification problem for the instance being processed.

To calculate which is the best K value for each new case, we use the Positive Predictive Value (PPV). The PPV is the proportion of instances which predicted to belong to certain class and are correctly classified. The Positive Predictive Value is defined as

$$PPV = \frac{\text{numberofTruePositive}}{\text{numberofTruePositive} + \text{numberofFalsePositive}} \quad (3)$$

where a True Positive is the event that the test makes a positive prediction, and the subject has a positive result. And a False Positive is the event that the test makes a positive prediction, and the subject has a negative result.

In a validation process, before the proper classifier is used, we calculate the PPV values for each predicted class, C_{Pr} , with different K values. As a result, for each C_{Pr} we obtain a PPV table which contains the different PPV values for each K value. Table 1 shows an example for a 3 class problem and K values from 1 to 4. We want to emphasize that the C_{Pr} is the class that a classifier predicted for the instance and not the class that the instance belongs to.

In order to make a deeper analysis of the classifier behavior and based on the results obtained for high values of K , we realize that it is not the same confidence

Table 1. Example of the PPV Tables, first version

| $C_{Pr}=1$ | | $C_{Pr}=2$ | | $C_{Pr}=3$ | |
|------------|---------|------------|---------|------------|---------|
| K | PPV | K | PPV | K | PPV |
| 1 | PPV_1 | 1 | PPV_1 | 1 | PPV_1 |
| 2 | PPV_2 | 2 | PPV_2 | 2 | PPV_2 |
| 3 | PPV_3 | 3 | PPV_3 | 3 | PPV_3 |
| 4 | PPV_4 | 4 | PPV_4 | 4 | PPV_4 |

that 8 of the 9 neighbors belong to the C_{Pr} or to belong 5. This is the reason why we have extended the tables shown in Table 1 and take in consideration not only the K value and the C_{Pr} ; we also take in consideration the amount of the cases that belongs to the C_{Pr} between the K neighbors, K_{Pr} . Hence, the tables that are used in classification task are extended to those shown in Table 2.

Table 2. Example of the PPV Tables, improved version

| $C_{Pr}=1$ | | | $C_{Pr}=2$ | | | $C_{Pr}=3$ | | |
|------------|----------|-----------|------------|----------|-----------|------------|----------|-----------|
| K | K_{Pr} | PPV | K | K_{Pr} | PPV | K | K_{Pr} | PPV |
| 1 | 1 | PPV_1 | 1 | 1 | PPV_1 | 1 | 1 | PPV_1 |
| 2 | 2 | PPV_2^2 | 2 | 2 | PPV_2^2 | 2 | 2 | PPV_2^2 |
| | 1 | PPV_2^1 | | 1 | PPV_2^1 | | 1 | PPV_2^1 |
| 3 | 3 | PPV_3^3 | 3 | 3 | PPV_3^3 | 3 | 3 | PPV_3^3 |
| | 2 | PPV_3^2 | | 2 | PPV_3^2 | | 2 | PPV_3^2 |
| | 1 | PPV_3^1 | | 1 | PPV_3^1 | | 1 | PPV_3^1 |
| 4 | 4 | PPV_4^4 | 4 | 4 | PPV_4^4 | 4 | 4 | PPV_4^4 |
| | 3 | PPV_4^3 | | 3 | PPV_4^3 | | 3 | PPV_4^3 |
| | 2 | PPV_4^2 | | 2 | PPV_4^2 | | 2 | PPV_4^2 |
| | 1 | PPV_4^1 | | 1 | PPV_4^1 | | 1 | PPV_4^1 |

Thus when a new case to be classified arrives, firstly we process it for different K values. For each K value, we get the C_{Pr} and the K_{Pr} and from the PPV tables we obtain the PPV value that correspond for each K . After that, we select the K with the highest PPV value and finally the K -NN result of this concrete K is assigned to the new case.

In Figure 2 we could see with an example how our algorithm works. In Figure 2(a) we could see the PPV tables that we obtain after a validation process. These tables show the real values that we got in our experiments with Ionosphere database. As it is a two-class problem there are two PPV tables, one for each C_{Pr} . On the other hand, in Figure 2(b) we could see the steps that our algorithm follows to classify a new case. In the first step, we get the 4 nearest neighbors classes. After that, in the second step, for different K values we apply the K -NN method obtaining the C_{Pr} and the K_{Pr} . In the third step, from the PPV tables

we obtain the PPV value which belongs to the K , C_{Pr} and K_{Pr} values. And finally we select the case with the highest PPV value, and we assign to the new case its C_{Pr} .

| $C_{Pr}=0$ | | | | | $C_{Pr}=1$ | | | | |
|------------|--------|----------|--------|----------------|------------|--------|----------|--------|----------------|
| | PPV | K_{Pr} | PPV | Numb. of Cases | | PPV | K_{Pr} | PPV | Numb. of Cases |
| K=1 | 0.8342 | 1 | 0.8342 | 199 | K=1 | 0.8689 | 1 | 0.8689 | 61 |
| K=2 | 0.8342 | 2 | 0.8865 | 185 | K=2 | 0.8689 | 2 | 0.9189 | 37 |
| | | 1 | 0.1429 | 14 | | | 1 | 0.7917 | 24 |
| K=3 | 0.8019 | 3 | 0.8956 | 182 | K=3 | 0.9167 | 3 | 0.96 | 25 |
| | | 2 | 0.2333 | 30 | | | 2 | 0.8696 | 23 |
| K=4 | 0.8341 | 4 | 0.9133 | 173 | K=4 | 0.9184 | 4 | 0.9474 | 19 |
| | | 3 | 0.3784 | 37 | | | 3 | 0.8947 | 19 |
| | | 2 | 1 | 1 | | | 2 | 0.9091 | 11 |

(a) PPV table for each C_{Pr} .

| | | | | | | | | |
|-----------|-----|----------|----------|-----|----------|---------------|-------|---|
| [0,1,1,0] | K | C_{Pr} | K_{Pr} | K | C_{Pr} | K_{Pr} | PPV | $K = 3$ PPV = 0.869 Final Class = 1 |
| | 1 | 0 | 1 | 1 | 0 | 1 | 0.834 | |
| | 2 | 0 | 1 | 2 | 0 | 1 | 0.142 | |
| | 3 | 1 | 2 | 3 | 1 | 2 | 0.869 | |
| 4 | 0 | 2 | 4 | 0 | 2 | 1*0.834=0.834 | | |

1. 4-NN Classes

2. Get C_{Pr} and K_{Pr} for each K

3. Get PPV value from PPV tables

4. Select the highest PPV

(b) Step by step

Fig. 2. In 2(a) we show the tables that our algorithm get in our experiments after a validation process with Ionosphere database. Each C_{Pr} table shows the PPV for different K , with different K_{Pr} . In 2(b) we show the steps that our algorithm follows to assign the final class

Sometimes it is possible to be few cases to achieve the PPV value. For example, in Figure 2(a), in $C_{Pr} = 0$ Table, we could see that when $K = 4$ and $K_{Pr} = 2$, there is only one case. As this case has been classified correctly in the validation process, its success rate is very high. This success rate is not very reliable because they do not have enough cases to support this claim, and it is possible that the case correctly classified in the validation process could be an exception. To avoid these cases we have included a threshold. If there is no more than 10 cases, the degree of confidence is multiplied by the degree of confidence of the K . We could see that in Figure 2(b) for $K = 4$ case.

5 Experimental Results

In this section we show the experimental results obtained with different databases. We have compared our method with K -NN for different K values.

5.1 Datasets

Twenty-six databases are used to test our hypothesis. All of them are obtained from the *UCI Machine Learning Repository* [20]. The characteristics of the databases are given in Table 3.

Table 3. The characteristics of the 26 databases used in this experiment

| Domain | Num. of Instances | Num. of Attributes | Num. of Classes |
|-----------------------|-------------------|--------------------|-----------------|
| Australian Credit | 690 | 14 | 2 |
| Balance | 625 | 4 | 3 |
| Blood Transfusion | 748 | 5 | 2 |
| Breast Cancer | 569 | 32 | 2 |
| Car | 1728 | 6 | 4 |
| Cmc | 1473 | 9 | 3 |
| Diabetes | 768 | 8 | 2 |
| Glass | 210 | 9 | 7 |
| Haberman | 306 | 3 | 2 |
| Image Segmentation | 2310 | 19 | 7 |
| Ionosphere | 351 | 34 | 2 |
| Iris | 150 | 4 | 3 |
| Letters | 20000 | 16 | 26 |
| Magic Gamma Telescope | 19020 | 11 | 2 |
| Optical Digits | 5620 | 64 | 10 |
| Pen-Based Digits | 10992 | 16 | 10 |
| Sonar | 208 | 60 | 2 |
| Spambase | 4601 | 57 | 2 |
| Statlog(German) | 1000 | 20 | 2 |
| Statlog (Heart) | 270 | 13 | 2 |
| Statlog (Landsat) | 6435 | 36 | 7 |
| Statlog (Shuttle) | 58000 | 9 | 7 |
| Tic Tac Toe | 958 | 9 | 2 |
| Vowel Context | 528 | 10 | 11 |
| Waveform | 5000 | 21 | 3 |
| Wine | 178 | 13 | 3 |

5.2 Experimental setup

In order to give a real perspective we have applied 5x2 fold cross validation to each database [21]. But firstly our algorithm needs a validation process to get the PPV values. So we have applied 5-fold out for each fold, where we have used the %70 as training and %30 as testing.

In Table 4 we show the results obtained by the K -NN method for different K values, where the best result is written in boldface. We do not include the $K=2$ value in this Table, because in the case of draw we give the preference to the nearest neighbor, and hence the $K=2$ and $K=1$ results always are the same.

In Table 5 we compare our method with the K -NN. Looking at the results of Table 5, it could be seen that in most of the cases our method improves

Table 4. Accuracy level percentage of the K -NN method

| Database | K -NN=1 | K -NN=3 | K -NN=4 | K -NN=5 | K -NN=6 | K -NN=7 | K -NN=8 | K -NN=9 | Avg | σ |
|-----------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|--------|----------|
| Australian Credit | 80.203 | 83.275 | 83.565 | 84.812 | 84.783 | 84.870 | 84.957 | 85.072 | 83.942 | 1.6561 |
| Balance | 78.814 | 81.571 | 83.109 | 84.808 | 86.571 | 87.436 | 87.660 | 87.917 | 84.736 | 3.3180 |
| Blood Transfusion | 68.797 | 73.396 | 73.743 | 75.374 | 75.802 | 76.337 | 76.417 | 77.406 | 74.659 | 2.7275 |
| Breast Cancer | 95.458 | 96.725 | 96.761 | 96.444 | 96.725 | 96.444 | 96.585 | 96.232 | 96.422 | 0.4296 |
| Car | 85.509 | 90.660 | 90.880 | 91.551 | 91.123 | 90.949 | 90.856 | 90.729 | 90.282 | 1.9483 |
| Cmc | 43.505 | 45.516 | 46.250 | 46.685 | 46.766 | 47.242 | 46.834 | 46.861 | 46.208 | 1.2078 |
| Diabetes | 69.474 | 74.323 | 73.308 | 75.602 | 74.023 | 75.489 | 75.451 | 75.789 | 74.182 | 2.1008 |
| Glass | 64.299 | 65.421 | 63.271 | 64.860 | 63.084 | 63.645 | 62.523 | 63.178 | 63.785 | 0.9878 |
| Haberman | 65.098 | 68.039 | 67.908 | 69.281 | 69.412 | 71.830 | 71.699 | 73.203 | 69.559 | 2.6199 |
| Image Segmentation | 95.792 | 94.909 | 94.814 | 94.035 | 94.424 | 93.844 | 94.147 | 93.706 | 94.459 | 0.6897 |
| Ionosphere | 85.486 | 84.171 | 84.400 | 83.771 | 84.171 | 83.600 | 83.600 | 82.743 | 83.993 | 0.7909 |
| Iris | 93.867 | 94.400 | 94.400 | 94.933 | 94.267 | 94.267 | 94.267 | 94.933 | 94.417 | 0.3595 |
| Letters | 94.343 | 94.291 | 94.422 | 94.016 | 94.098 | 93.647 | 93.609 | 93.217 | 93.955 | 0.4251 |
| Magic Gamma Telescope | 80.124 | 82.211 | 82.462 | 82.980 | 83.115 | 83.212 | 83.281 | 83.213 | 82.575 | 1.0636 |
| Optical Digits | 98.359 | 98.456 | 98.491 | 98.349 | 98.406 | 98.185 | 98.256 | 98.139 | 98.330 | 0.1263 |
| Pen-Based Digits | 99.212 | 99.128 | 99.148 | 98.970 | 99.034 | 98.810 | 98.866 | 98.675 | 98.981 | 0.1859 |
| Sonar | 82.788 | 77.692 | 79.423 | 74.519 | 75.385 | 70.192 | 72.019 | 68.173 | 75.024 | 4.8764 |
| Spambase | 88.530 | 88.965 | 89.583 | 88.757 | 89.435 | 88.596 | 89.178 | 88.130 | 88.897 | 0.4893 |
| Statlog (German) | 66.960 | 69.200 | 68.840 | 70.000 | 70.520 | 70.620 | 71.580 | 71.000 | 69.840 | 1.4723 |
| Statlog (Landsat) | 89.473 | 90.002 | 89.983 | 89.890 | 90.045 | 89.557 | 89.765 | 89.442 | 89.770 | 0.2478 |
| Statlog (Heart) | 75.778 | 78.963 | 78.889 | 81.037 | 80.222 | 80.370 | 80.519 | 80.593 | 79.546 | 1.7060 |
| Statlog (Shuttle) | 99.932 | 99.876 | 99.880 | 99.835 | 99.835 | 99.794 | 99.792 | 99.751 | 99.837 | 0.0581 |
| Tic-tac-toe | 100.000 | 99.311 | 99.478 | 97.745 | 98.455 | 96.013 | 97.641 | 95.177 | 97.978 | 1.6995 |
| VowelContext | 93.758 | 80.202 | 76.687 | 63.737 | 59.758 | 49.394 | 44.626 | 38.646 | 63.351 | 19.1055 |
| Waveform | 77.160 | 80.448 | 80.348 | 81.748 | 81.476 | 82.696 | 82.504 | 83.044 | 81.178 | 1.9020 |
| Wine | 94.719 | 95.506 | 95.843 | 95.506 | 95.730 | 96.180 | 96.292 | 96.742 | 95.815 | 0.6117 |

the mean of all the K , only getting worse results in 5 databases. This can be considered logical because in Table 4, it can be seen that in some databases there are considerable differences for different values of K , which makes to devalue the mean.

On the other hand, if we compare our method with the best K , we could see that it improves in 6 databases, it draws in 1 and get worse in 20. Although this results do not seem promising, looking in more detail, we could see that the majority of the improved databases have low standard deviation value, σ , for the different K values. There are 12 databases which has $\sigma < 1$, and the results are very similar; in 5 of them our method gets the best result, in 1 they draw and in the other 6 the best K -NN is the optimal. We think that these are optimistic results taking into account that we are comparing our methods only result with the best between the K -NN's 9 results.

6 Conclusion and Further Results

A new method extending the K -NN idea is presented in this work: PPV K -NN. The main reason of this approach is to ensure that the final decision is made with some confidence. For doing that we search the most reliable K for each new case.

The new method has been implemented and tested over 26 databases from the UCI repository. We have compared our method with the best result and the mean of K -NN. Getting interesting results for the databases with regular results for different K in K -NN method.

Table 5. Comparison between our method resultd with K -NN's best result and mean

| Database | PPV K -NN | K -NN Best | Avg K -NN |
|-----------------------|---------------|----------------|-------------|
| Australian Credit | 84.116 | 85.072 | 83.942 |
| Balance | 86.635 | 87.917 | 84.736 |
| Blood Transfusion | 76.364 | 77.406 | 74.659 |
| Breast Cancer | 96.655 | 96.761 | 96.422 |
| Car | 90.197 | 91.551 | 90.282 |
| Cmc | 47.541 | 47.242 | 46.208 |
| Diabetes | 73.947 | 75.789 | 74.182 |
| Glass | 64.766 | 65.421 | 63.785 |
| Haberman | 72.484 | 73.203 | 69.559 |
| Image Segmentation | 95.446 | 95.792 | 94.459 |
| Ionosphere | 88.286 | 85.486 | 83.993 |
| Iris | 94.133 | 94.933 | 94.417 |
| Letters | 94.659 | 94.422 | 93.955 |
| Magic Gamma Telescope | 82.387 | 83.281 | 82.575 |
| Optical Digits | 98.491 | 98.491 | 98.330 |
| Pen-Based Digits | 99.216 | 99.212 | 98.981 |
| Sonar | 79.904 | 82.788 | 75.024 |
| Spambase | 89.561 | 89.583 | 88.897 |
| Statlog (German) | 70.700 | 71.580 | 69.840 |
| Statlog (Heart) | 79.778 | 81.037 | 79.546 |
| Statlog (Landsat) | 90.228 | 90.045 | 89.770 |
| Statlog (Shuttle) | 99.924 | 99.932 | 99.837 |
| Tic-tac-toe | 99.937 | 100.000 | 97.978 |
| VowelContext | 92.667 | 93.758 | 63.351 |
| Waveform | 80.720 | 83.044 | 81.178 |
| Wine | 96.966 | 96.742 | 95.815 |

As future work it would be interesting to include the reject option. Where we reject the cases where there are two different K with a high PPV value that predict different classes.

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